Extended Geometry for Electrical Engineers

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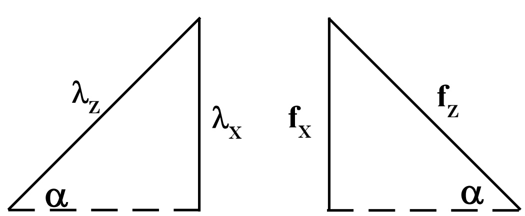
Traditional geometry provides a basis for understanding how mathematics can be used to identify relationships between elements of geometric figures. Electrical engineers deal with wavelengths and frequencies and the relationship between these two different types of "quantity of something", but conventional geometry, using right triangles, does not provide a methodology for representing the relationships between two different quantity of things that have an inverse proportioanl relationship.

Frequency (f) is defined as the periodicity of some type of event. Mathematically, the periodicity of waves are presented as $f = 1 / \tau$, where τ is how many times each full period of a wave occurs in one unit of time. Wavelength (λ) is defined as the distance between the peaks of a wave. Algebraically, the relationships between wavelength and frequency are expressed as below:

$$f = c / \lambda \qquad \lambda = c / f \qquad c = f * \lambda \qquad (1)$$

Wavelength is fairly straight forward, its size can be represented by any numeric value, and its unit designator, meters, centimeters or other accepted unit of length, has a single descriptor. Although frequency is conventionally expressed by a single descriptor, Hertz (Hz), it is actually a complex descriptor, $Hz = 1\ s^{-1}$, or $Hz = 1 / \tau$, and was expressed as cycles per second (cps) before the term Hertz (Hz) was defined. It was found that frequency had to have a value of 2π to represent a frequency in the frequency triangle. Although angular frequency can have a value of 2π when frequency has a value of 1, the unit designators are radians per second rather than cycles per second. A proof has not been identified as to why frequency has to have a numeric value of 2π, but herein it will be described as an "intrinsic frequency." An equivalent unit value for a wavelength would have a value of 1, an "intrinsic wavelength," but this does not represent a physical size in a current system of measurement.

Applying some numeric value to the legs of a triangle is conventional, and if both numbers represents wavelengths, or frequencies, the relationships still hold true. What is not conventional is pairing two triangles with different unit descriptors. Figure 1 represents the pairing of two triangles, one representing wavelength and the other frequency. By themselves, the two triangle forms are conventional, but they are related to each other in a way that is not intuitive. Their relationship can be shown mathematically. Only one leg is needed (they have to be the same leg position) to represent the relationship.



Wavelength **Frequency**

Figure 1.

The angle for each triangle will be the same, but it does not have to be 45 degrees.

When the cross products are equal it indicates that the wavelength and frequency values are mutually related, one is the inverse of the other. The sum of the cross products is the constant of proportionality (k) and value derived, by either equation (2) or (3) below, is expressed in length, L, per "unit of time", τ (tau).

$$\mathbf{k} = \lambda_X * \mathbf{f}_Z \qquad (2)$$

$$\mathbf{k} = \lambda_Z * \mathbf{f}_X \qquad (3)$$

The constant of proportionality between wavelength and frequency is the speed of light (SOL). The intrinsic unit values of a wavelength and frequency can be applied to the triangles, giving Figure 2.

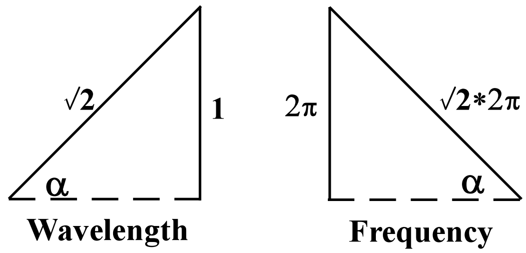


Figure 2.

Applying the intrinsic values as expressed in Figure 2 to equations (2) and (3) produce the results in equations (4) and (5).

$$\mathbf{k = \lambda_X * f_Z = 1 * (\sqrt{2} * 2\pi) = \sqrt{2} * 2\pi} \qquad \mathbf{L/\tau} \qquad (4)$$
$$\mathbf{k = \lambda_Z * f_X = 2\pi * \sqrt{2}} \qquad \mathbf{L/\tau} \qquad (5)$$

Using the cosecant trigonometric notation, equations (2) and (3) would be expressed as in equations (6) and (7) when the angle (α) is 45 degrees.

$$\mathbf{k = \lambda_X * (f_X * \csc(\alpha)) = 1 * (2\pi * \sqrt{2}) = \sqrt{2} * 2\pi} \qquad \mathbf{L/\tau} \qquad (6)$$
$$\mathbf{k = (\lambda_X * \csc(\alpha)) * f_X = \sqrt{2} * 2\pi} \qquad \mathbf{L/\tau} \qquad (7)$$

The numeric values used are the intrinsic values that apply to wavelength and frequency. The actual size of the wavelength nor that of the duration of the unit of time are known, thus frequency is generic. It is apparent, when expressed in equations (6) and (7), that the unit of time (τ) is a function of the angle, as the constant of proportionality will change with the angle because the intrinsic unit sizes, based upon the vertical legs of Figure 2, do not change.

We know the value of the constant of proportionality (SOL) when expressed in meters and the second, 299,792,458 m s^{-1}. Using an assumption that frequency is scaled by factor of 10^8, applying this to 2π, and dividing that value into the SOL will give a value of 47.713 cm for the hypotenuse of the wavelength triangle. The physical size of the vertical leg of the wavelength triangle is not known nor the numeric magnitude of the hypotenuse of the frequency triangle, but we know multiplying these two numbers together will give the value for the SOL; an iteration process can be used to establish the two values, giving the results shown in Figure 3.

The actual physical size of the "intrinsic wavelength" is 21.106 cm. Applying equations (2) and (3) indicates that the cross product produces the known value for the SOL at the given angle. The two triangles are inversely proportional. The relationships shown in Figure 3 have significance in equations used to define physical law, and the manner in which units of measure should be defined..

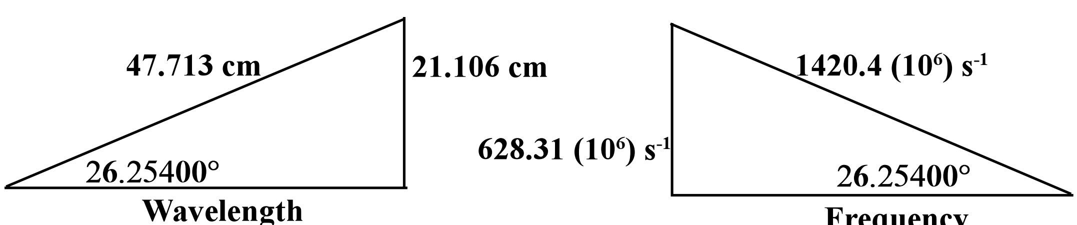


Figure 3.