The Basis For Wavelength And Frequency Right Triangle Geometric Pairs
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Electromagnetic Primitive Units

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Abstract: Wavelengths and frequencies can be expressed as the elements of geometric forms. Wavelengths and frequencies cannot be mixed in the same geometric form, however, specific pairs of geometric forms are related when they represent their mutual inverse relationships. A right triangle interjects the inverse relationship into the geometric relationship such that a parameters of the inverse relationship become a function of the angle. In the fundamental right triangle pair, wavelength and frequency must be applied in their "primitive units". For electromagnetic (EM) waves, the geometric and inverse relationships are readily demonstrated with triangle pairs using "primitive units", "practical units" and "System International (SI) units".

Electromagnetic Primitive Units

A primitive "unit wavelength" can be expressed as a dimensionless number, a unit, or a value of "one". A primitive "unit frequency" is expressing a "unit wavelength" in a "unit of time", which is a rate. One cycle of a wave in one unit of time can be expressed as an angular value when it is described in radians, and it would have the value of 2π. That primitive frequency has the value of 2π is not intuitive and its mathematical validity is demonstrable only when the primitive units for wavelength and frequency are used as the legs of right triangles. The primitive "unit frequency" has the duality of being both a frequency and an angular frequency, and this is valid for any tens multiple or division of 2π.

The geometric depiction of the use of the primitive values for wavelength and frequency are illustrated in Figure 1. The value of the constant of proportionality, "k", is the cross product of the hypotenuse of one triangle with either leg of the other triangle.

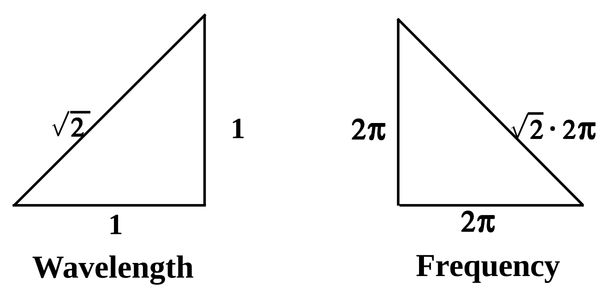

Figure 1.

The constant of proportionality is calculated by the values of Y and Z as shown in Figure 2. When the cross products are equal it indicates that the wavelength and frequency values shown by the triangle pairs are mutually related, one is the inverse of the other. The constant of proportionality is expressed by either equation (1) or (2).

$$k = YW * ZF = 1 * (\sqrt{2} * 2\pi) = \sqrt{2} * 2\pi \qquad (1)$$

$$k = YF * ZW = 2\pi * \sqrt{2} \qquad (2)$$

It should be noted that the constant of proportionality can be calculated for any angle when one leg of each of the triangle pairs is retained as a fixed value. When ZW or ZF are expressed as a trigonometric function, the cosecant function was chosen, the values can be expressed for any angle.

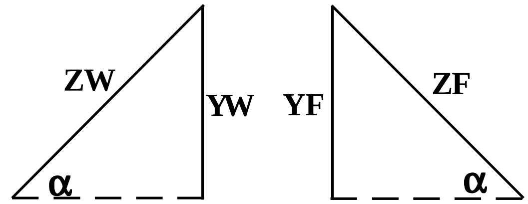

Figure 2.

$$ZW = YW * CSC(\alpha) \qquad \text{or} \qquad ZF = YF * CSC(\alpha) \qquad (3)$$

In determinate form, when the cross products are zero, a triangle pair are mutually related, one represents the inverse of the other.

$$\begin{vmatrix} YW & ZW \\ YF & ZF \end{vmatrix} = YW * ZF - ZW * YF = 0 \qquad (4)$$

It should be noted that the value of k can be calculated to near unlimited precision when the angle is at 45 degrees. The value of k is dimensioned as a “unit wavelength” in a “unit of time”. The “unit wavelength” has an actual physical length, and the “unit of time” has an actual duration, but these values are not determinable when expressed at the primitive unit level.

Practical Scaling

The scaling can be applied to the primitive “unit frequency”, which will in turn be reflected in the physical size of the “unit wavelength”. For practical use, the numeric values for electromagnetic wave frequencies are scaled by multiples to keep the numeric values within a reasonable size, which are typically a million or more cycles in a time duration of just a second. It is known that the desired scaling factor will be 10^8, however, the decimal point will be shifted two points to the right in order to represent conventional notation when the values are numerically presented.

Applying the scaling factor to the primitive values of Figure 1 results in the values shown in Figure 3. The “unit wavelength” retains the value of one, but its actual physical size is changed by the inverse of the tens factor applied to the frequency. The actual physical length of the “unit wavelength” will not be known until it is compared to an existing unit of measurement.

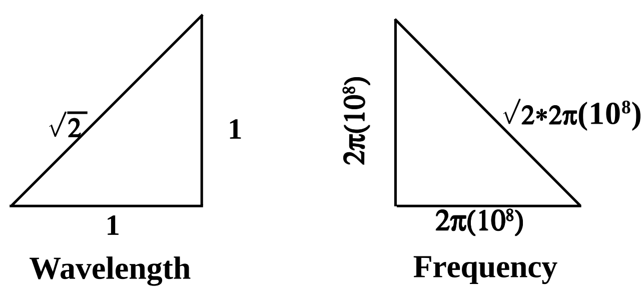

Figure 3.

The proportionality constant remains the same and can be calculated using either equation (1) or (2), giving it the same numeric value as before but having a value scaled to a practical electromagnetic frequency range. The unit designators remain the same, giving 888.5765 (10^6) “unit wavelengths” in a “unit of time”. The duration of the “unit of time” will not be apparent until the “dimensions” are expressed in SI units. Within mathematical equations the fundamental form $\sqrt{2}*2\pi(10^8)$ would provide some degree of simplification.

The constant of proportionality will vary as a function of the angle, which is why it is identified by the letter “k” to distinguish it from the letter “c” which is associated with a fixed value in SI units.

SI Units

The wavelength and frequency values of the triangle pair can be displayed in SI units, however, there is only one angle where the SI “unit of time”, the second, is valid. The numeric value of the wavelength associated with the primitive “unit frequency” is known to a limited precision in SI units. Using the frequency scaling factor shown in Figure 3, the value is 47.713.... cm. The hypotenuse of the wavelength triangle represents the inverse of the fixed leg in the frequency triangle. The 47.713 hypotenuse value will be related to the real physical value of the fixed leg in the wavelength triangle when the triangle pairs reflect their mutual inverse relationships in SI units. An iterative process can be used to determine the physical value of the "unit wavelength" in SI units. Figure 4 denotes the numeric values of the wavelength and

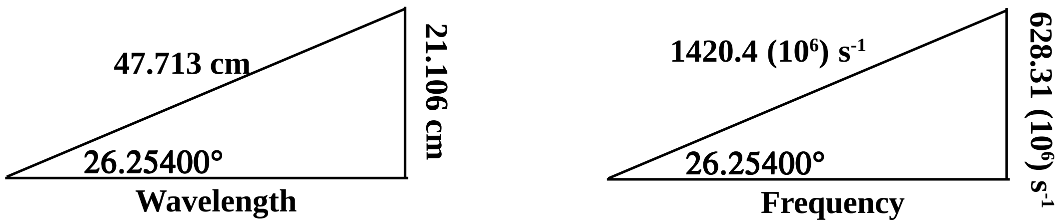

Figure 4.

frequency legs, and the hypotenuses when the angle represents the value associated with the time duration of the second.

The numeric value for the "unit wavelength", its associated frequency, the hypotenuse of the frequency triangle, and the angle can be calculated only to the precision permitted by SI units. When the "unit frequency" is scaled to practical EM values it can be determined that the "unit wavelength" is associated with the precession emission of neutral hydrogen, often referred to as the H1 hyperfine emission.

The duration of the "unit of time" is a function of the angle. When the duration of the "unit of time" is assigned a value of "one" at 45 degrees, the time duration at other angles is given by the ratio of the cosecant values, equation (5).

$$R = CSC(\alpha)/CSC(45) \qquad (5)$$

The duration of the second is approximately 1.5985... longer than the "unit of time" defined mathematically at the angle of 45 degrees.

The velocity of a EM wave has the same numeric value as the frequency when the unit of length is that of the wavelength. When defined at 45 degrees, a reference value for the speed of light is calculable to near unlimited precision. At 45 degrees, the triangles are at the geometric mathematical null point for the relationship between wavelength and frequency, both legs are equal.

The duration of the "unit of time" has a unity value when defined at 45 degrees. Clocks and electronic counters that emulate the duration of the "unit of time" will be limited in precision only by our technical capability to produce these devices.

To put the concept presented above into perspective, the conventional non-geometric form which uses the constant of proportionality is shown by the familiar three equation forms below, where **f** is frequency, $\boldsymbol{\lambda}$ is wavelength, and **c** is the speed of light, these defined using SI units.

$$\mathbf{f = c / \lambda \quad or \quad \lambda = c / f \quad or \quad c = f * \lambda} \qquad (6)$$

The difference is that SI units are defined by starting with a "unit of time", and defining the other units based upon that duration. The concept presented herein mutually defines the "unit of time" and the "unit of length" knowing they are related to the speed of light.