

A Mathematical Process to Derive the Velocity of Light Using Mutually Defined Units

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The current numeric value for the speed of light was determined by measuring its velocity in terms of a predefined unit of length and duration of time. The units defined by System International (SI) have proven useful in providing a uniform measurement system for scientific purposes, but they have limited precision and their basis is geocentric. The Consultative Committee on Units (CCU) stated, “the consensus that now exists on the desirability of finding ways of defining all of the base units of the SI in terms of fundamental physical constants so that they are universal, permanent and invariant in time”[1]. A mathematical process has been identified that allows the numeric value for the speed of light to be mutually derived along with the units that describe its velocity. The process exploits the known relationship between electromagnetic wavelengths and frequencies when it is expressed in a geometric form. The resultant numerical value for the speed of light and its associated units are based upon fundamental physical and mathematical constants, and they have near unlimited precision.

I. Intrinsic Units

This document is concerned with the mathematical derivation for a numeric value for the velocity of the speed of light, thus the material will be referring to electromagnetic wavelengths and frequencies.

Classical physics defines a wavelength as the distance between successive crests of a wave and it has a generic length with a value of 1 without an associated unit of measure. This same length can represent the circumference of a circle with a value of 1. In its circular form, the distances around the circle represent angular values. This combination wavelength/angular numeric value is defined as an “intrinsic wavelength”.

Frequency is defined as the number of wave crests that pass through a given point in a specified period of time. Mathematically, frequency is expressed as the inverse of T, the duration of time, $f=1/T$. In the general definition, T can be any defined duration. Frequency can be expressed in angular form and it has the mathematical form $\omega=2\pi/T$. There is one unique numeric value where frequency and that of angular frequency have the same numeric value, 2π . That numeric point of coincidence is valid for any tens multiple or division of 2π . This combination frequency/angular frequency numeric value is defined as an “intrinsic frequency”.

The algebraic equation that identifies the inverse

proportional relationship between wavelength and frequency has three forms, equation (1), where f is frequency, λ is wavelength and c the speed of light. The constant of proportionality is the value of c.

$$f=c/\lambda \quad \lambda=c/f \quad c=\lambda*f \quad (1)$$

The above algebraic equations do not provide a mechanism to accommodate numeric values that have dual meanings. The duality is fully exploited when intrinsic wavelengths and frequencies are expressed as the elements of a right triangle.

II. Wavelength Frequency Triangle Pair

There are no mathematical reasons why wavelengths and frequencies cannot be used to define the elements of a right triangle, Figure 1, but they cannot be mixed in the same triangle. When the two intrinsic values are used to define the basic elements of their respective right triangles, their geometric re-

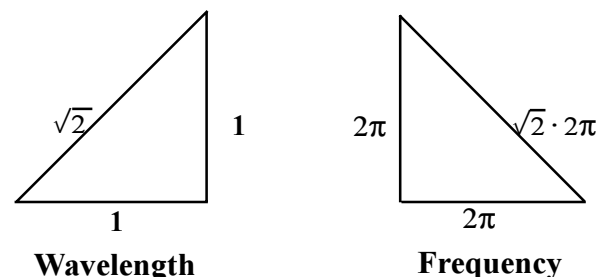


Figure 1.

relationships become a function of the angle. The duality of the intrinsic values create the angular synchronization of the wavelength frequency triangle pair.

For the triangle pair the constant of proportionality is the cross product of the hypotenuse of one triangle with either leg of the other triangle. The symbol k will be used for the constant of proportionality to distinguish it from the symbol c , which is commonly used to represent a specific value using SI units. The constant of proportionality can be calculated by the values of Y and Z shown in Figure 2.

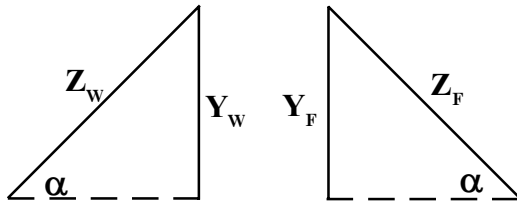


Figure 2.

When the cross products are equal it indicates that the wavelength and frequency values calculated using the triangle pair relationships are mutually related, one is the inverse of the other. The constant of proportionality is expressed by either equation (2) or (3), which are expressed in lengths, L , per “unit of time”, τ (tau).

$$k = Y_W * Z_F = 1 * (\sqrt{2} * 2\pi) = \sqrt{2} * 2\pi L/\tau \quad (2)$$

$$k = Y_F * Z_W = 2\pi * \sqrt{2} L/\tau \quad (3)$$

It should be noted that the constant of proportionality can be calculated for any angle when one leg of each of the triangle pair is retained as an intrinsic value. The time unit has a value of 1 when the angle is at 45 degrees, however, its actual duration and that of the actual physical length of L are indeterminate at the intrinsic level.

III. Practical Units

The wavelength frequency inverse proportionality does not change when the frequency is scaled. When scaled, the intrinsic frequency mathematically creates an intrinsic wavelength with a length associated with the scaled frequency. Even though the in-

trinsic wavelength has been scaled to a practical wavelength, its numeric value is still expressed as $1L$ (one length). From a human perspective a length has to have a reasonable physical length for practical use, but the scaling process produced a result that was unexpected.

Figure 3 illustrates the triangle pair values when the frequency has been scaled by 10^8 . The leg of the frequency triangle has a numeric value of $628.31...(10^6)$ $c\tau$ (cycles per unit time). The result-

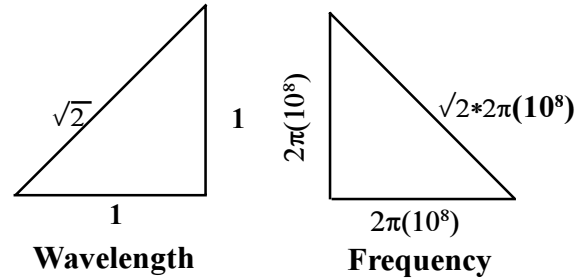


Figure 3.

ant numeric value for the constant of proportionality is $888.5765...(10^6) L/\tau$.

The hypotenuse of the triangle represents either the wavelength or frequency of the opposite triangles legs. In the case of the wavelength and its associated frequency, the numeric value of the frequency has the same numeric value as the speed of light.

Even though the frequency has been scaled by a factor of 10^8 , that does not provide, to the human perception, the physical length of L or the duration of the time unit τ . These will not be determinable until the value of L and τ are related to a familiar system of measurement, i.e., SI units.

IV. SI Units

The wavelength and frequency values of the triangle pair can be displayed in SI units, however, there is only one angle where the SI “unit of time”, the second, is valid. The vertical leg was chosen to be the triangle pair element that will use their respective fixed scaled intrinsic values. The numeric value of the wavelength associated with the intrinsic frequency is known to a limited precision in SI units. The hypotenuse of the wavelength triangle represents the wavelength of the vertical leg of the frequency triangle. When the vertical leg frequency is expressed

in SI units, it will give the wavelength in SI units. Using the middle formula of equation (1), and the frequency scaling factor shown in Figure 3, gives a value of 47.713... cm.

An iteration process can be used to determine the angle that will give that value for the hypotenuse of the wavelength triangle. Figure 4 denotes the numeric values of the wavelength and frequency legs, and the hypotenuses, when the angle represents the value associated with the time duration of the second. When expressed in SI units the relationships between the triangle pair are more readily visualized.

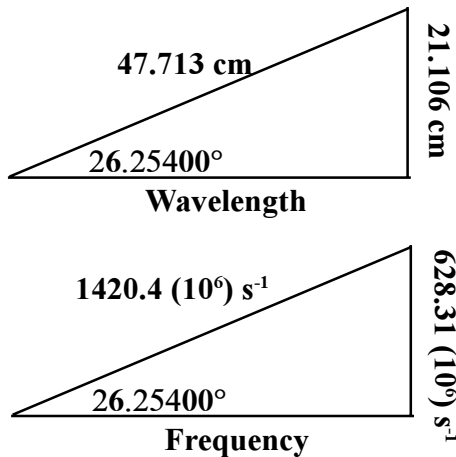


Figure 4.

The actual physical length of the scaled intrinsic wavelength was not known until it was extracted in a manner that allowed it to be compared to an existing system of measurement.

The scaled intrinsic frequency results in a scaled intrinsic wavelength that is that of the precession emission of neutral hydrogen, usually referred to as the H1 hyperfine emission.

In SI units, the numeric value for the practical intrinsic wavelength, its associated frequency and the angle, can be calculated only to the precision permitted by that system.

The duration of the “unit of time” is a function of the angle. When the duration of the “unit of time” is assigned a value of “one” at 45 degrees, the time duration at other angles is given by the ratio of the cosecant values, equation (4).

$$R_{\tau} = \text{CSC}(\alpha)/\text{CSC}(45) \quad (4)$$

The duration of the second is approximately 1.5985... longer than τ .

V. Conclusions

The numeric value for the speed of light was derived without knowing the physical values of the units by which it is defined. The physical values for the units were mutually derived along with a numeric value for the speed of light.

In this mathematical process it is significant that the velocity of an electromagnetic wave has the same numeric value as its frequency. When defined at 45 degrees, a reference value for the speed of light is calculable to near unlimited precision.

The duration of the time unit can be expressed in terms of the frequency of the reference emission, which is $\sqrt{2} * 2\pi(10^8)$ Hz.

The frequency of the reference emission is used to establish the basis for a “energy unit”.

The use of the mathematical form for the speed of light value, $\sqrt{2} * 2\pi(10^8)$, will make it convenient to use in general scientific formula.

The mathematically derived reference value for the speed of light provides a numeric form that is precisely suited for mathematical formulations used to determine the theoretical basis of physical law. The real value of the speed of light and other forms of electromagnetic emissions can be expected to vary from the mathematically extracted value dependent upon the characteristics of the medium in which it is permitted to propagate.

Using the values derived by this process will allow the creation of a set of scientific units that are universal, permanent and invariant in time.

References

- [1] <http://www.iupap.org/commissions/interunion/iu1/u1-2005.pdf>