

Time, Space and Energy

The Mathematical Convergence of Their Units of Measure

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Abstract: The terms time, space and energy represent fundamental characteristics of the physical universe, however, the sizes assigned to the units are arbitrary and do not exploit their common link. To be valid for scientific inquiry, the base units of measure should be non-arbitrary and mutually defined based upon fundamental physical law. A simple geometric based mathematical process can be used to extract a set of base units for time and space that are mutually defined based upon physical law, and which will simultaneously identify the basis for a unit of energy. A numeric value is identified that provides the mathematical convergence of the units which describe time, space and energy.

I. Consensus

The 2005 final report of the Consultative Committee on Units (CCU) contained the following statement, “the consensus that now exists on the desirability of finding ways of defining all of the base units of the SI in terms of fundamental physical constants so that they are universal, permanent and invariant in time;”⁽¹⁾

The base units of measure defined by the System International (SI) are the second, meter, kilogram, ampere, kelvin, mole and candela. The SI does not identify a term for a base unit of measure for energy.

There are several basic problems that must be addressed to achieve the CCU’s consensus. The first issue is the tendency to define a base unit of measure in terms of human perception rather than basing it on a fundamental physical constant. A base unit defined in terms of a fundamental physical constant may not be suitable for commercial and everyday use, but this can be corrected by providing a derived definition.

Fundamental physical constants are, in the strictest sense, physical constants that are independent of systems of units. However, what the SI is actually accomplishing is defining a dimension and applying a given size to a fundamental characteristic of the physical universe such that it can be used as the basis for a base unit of measure.

II. Natural Link

Ideally, all the base units of measure should have a natural link to some common element. This would be difficult to do for a base unit that will be associated with some physical condition related to the earth, such as mass, but it can be achieved for time, energy and space (length or distance).

Figure 1 illustrates the ideally linked base units for time, energy and length. The meter and the second could have been easily defined to have a common link, but those that created the meter

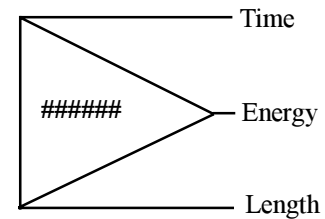


Figure 1 - Time, Energy, Length

some 200 years ago were unaware of the existence of electromagnetic waves, nor did they know that light was an electromagnetic phenomena, or that frequency and energy were associated. We know enough today to identify the common link between time, energy and length.

III. Definitions

The current definitions for time and length have a common element, but the way they are presented masks this commonality. Time, length and energy have frequency equivalent definitions. The official definitions of the second and meter illustrate their frequency equivalents.

“The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom.” This transition count represents 9192.631770 MHz.

“The meter is the length of the path traveled by light in vacuum during a time interval of 1/299 792 458 of a second.” A wavelength of one meter in length represents a frequency of 299.792458 MHz.

Energy is dimensioned using the joule and it is not a SI base unit. The joule is a relatively large amount of energy and its frequency equivalent would be extremely large. An eV was created to express energy at the atomic level and it is equated to a length of 1,239.8424121 nm (nanometers), which represents a frequency of 241,798.8407835012 GHz. That value is quite large even when related to the frequency equivalency of the second.

The time and energy frequency representations do not have values that can be identified as a fundamental physical constant. The length frequency representation is associated with a fundamental physical constant but its current value is arbitrary. As illustrated by Figure 1, a frequency that is mutually derived to represent time, length and energy would provide the basis for a scientific set of base units.

IV. Wavelength Frequency Relationship

This document is concerned with the mathematical processes that will involve a numeric value for the velocity of the speed of light (SOL), thus the material will be referring to electromagnetic wavelengths and frequencies.

The classical method of expressing the relationship between wavelength and frequency is with the algebraic expression,

$$\mathbf{f=c/\lambda \quad \lambda=c/f \quad c= \lambda*f \quad (1)}$$

where f is frequency, λ is wavelength and c the speed of light, the constant of proportionality between wavelength and frequency. c, in the algebraic expressions, is based upon a defined unit of length and a fixed time unit duration, currently the SI second.

The wavelength frequency relationship can be expressed geometrically using a pair of triangles, one

representing wavelength and the other frequency, Figure 2, but this is a special case that replicates the equation (1) processes in a geometric form.

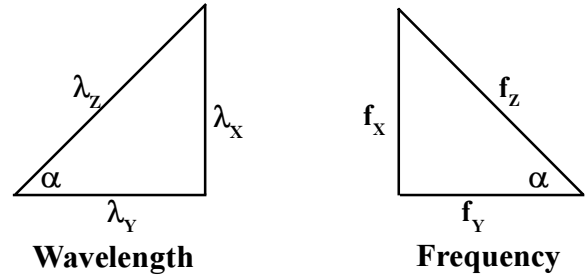


Figure 2.

The wavelength frequency relationships illustrated by Figure 2 can be expressed in an algebraic form similar to that of equation (1).

$$\mathbf{f_z = c_\alpha / \lambda_x, \quad \lambda_z = c_\alpha / f_x, \quad c_\alpha = \lambda_x * f_z \quad (2)}$$

or

$$\mathbf{f_x = c_\alpha / \lambda_z, \quad \lambda_x = c_\alpha / f_z, \quad c_\alpha = \lambda_z * f_x \quad (3)}$$

The values of λ_y and f_y can be used instead of λ_x and f_x , but this text will utilize the vertical leg of each triangle to establish the mathematical relationships between the two triangles. The prime difference between the equations sets (2) and (3) versus that of equation set (1) is that the value of c is based upon a function of the angle of the right triangle. c does not appear in the geometric form directly, it is the product of wavelength and frequency, the same as in the last equation of equation set (1). However, in SI units, the SOL is specifically defined to be a function of a length in relationship with the SI second. The value of c obtained as a cross product between the two triangles is defined by the geometric characteristics of the two triangles and the values of the legs.

The geometric form illustrated by Figure 2 and equation sets (2) and (3) are much more useful than equation set (1), but it requires identifying the values of λ_x , λ_y , f_x and f_y such that they will result in hypotenuse values, in the opposite triangle, that reflect the inverse proportional relationship between wavelength and frequency.

V. Wavelength Frequency Intrinsic Units

Classical physics defines a wavelength as the distance between successive crests of a wave and it has a generic length with a value of 1, without an identified measure of length. This wavelength numeric value is defined herein as an “intrinsic wavelength.” A wavelength is dimensioned with a single unit which depicts size.

Frequency is defined as the number of wave crests that pass through a given point in a specified period of time. A single unit of frequency can be expressed in linear form as having a value of 1, but for it to be useable in the frequency triangle it must be represented in its angular form, which has a numeric value of 2π . The value of this angular period, without an identified period of time, is defined herein as an “intrinsic frequency.” Frequency is dimensioned with two units which depicts a rate of occurrence.

VI. Constant of Proportionality

When the two intrinsic values, for wavelength and frequency, are used to define the basic elements of their respective right triangles, the SOL becomes a function of the angle.

For the triangle pair, Figure 2, the constant of proportionality is the cross product of the hypotenuse of one triangle with either leg of the other triangle. The symbol k will be used for the constant of proportionality to distinguish it from the symbol c , which is commonly used to represent a specific value for the SOL using SI units.

The constant of proportionality can be calculated using the symbolic dimensions of wavelength and frequency, shown in Figure 2, and the numeric values shown in Figure 3.

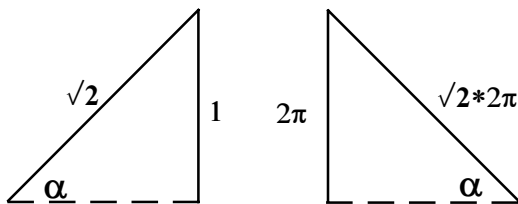


Figure 3.

$$k = \lambda_x * f_z = 1 * (\sqrt{2} * 2\pi) = \sqrt{2} * 2\pi \text{ L}/\tau \quad (4)$$

$$k = \lambda_z * f_x = \sqrt{2} * 2\pi \text{ L}/\tau \quad (5)$$

When the cross products are equal, it indicates that the wavelength and frequency values calculated using the triangle pair relationships are mutually related, one is the inverse of the other. The constant of proportionality is expressed by either equation (4) or (5), which is expressed by length, L, and per “unit of time”, t (tau).

It should be noted that the constant of proportionality can be calculated for any angle when one mutual leg of each of the triangle pair are retained at their intrinsic value. The time unit, tau, has a value of 1 when the angle is at 45 degrees, however, its actual duration, and that of the physical length of L, are indeterminate strictly at the intrinsic level. These values can be determined when the angle that represents the SI second is identified.

VII. Practical Units

The wavelength frequency inverse proportionality does not change when the frequency is scaled. When scaled, the intrinsic frequency mathematically creates an intrinsic wavelength with a length associated with the scaled frequency. Even though the intrinsic wavelength has been scaled to a practical wavelength, its numeric value is still expressed as 1L (one length). The actual physical size of L will be dependent upon the scaling factor of the intrinsic frequency, and, at this point in the mathematical process, its numeric size, relative to a known unit of length, is unknown.

Figure 4 illustrates the triangle pair values when the frequency has been scaled by 10^8 . The leg of the frequency triangle has a numeric value of $2\pi(10^8)$ c τ (cycles per unit time). The result-

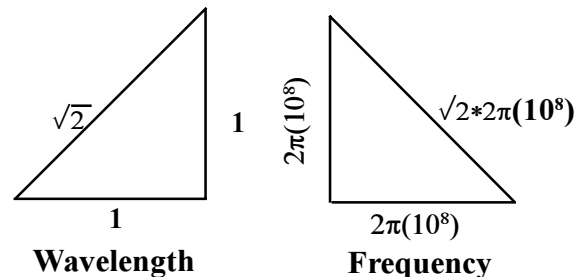


Figure 4.

ant numeric value for the constant of proportionality, the SOL, is $888.5765... (10^6) L/\tau$.

The hypotenuse of each triangle represents either the wavelength or frequency of the opposite triangles vertical leg. In the case of the wavelength leg and its associated frequency, the numeric value of the frequency has the same numeric value as the SOL.

Even though the frequency has been scaled by a factor of 10^8 , that does not provide, to the human perception, the physical length of L or the duration of the time unit τ . These will not be determinable until the value of L and τ are related to a familiar system of measurement, i.e., SI units.

VIII. Wavelength Frequency Triangles-SI Units

The wavelength and frequency values of the triangle pair can be displayed in SI units, however, there is only one angle where the SI “unit of time”, the second, is valid. The vertical leg was chosen to be the triangle pair element that will use their respective intrinsic values. The numeric value of the wavelength associated with the intrinsic frequency is known to a limited precision in SI units. The hypotenuse of the wavelength triangle represents the wavelength of the vertical leg of the frequency triangle. When the vertical leg frequency is expressed in SI units, it will give the wavelength in SI units. Using the middle formula of equation (1), and the frequency scaling factor shown in Figure 4, gives a value of $47.713... \text{ cm}$. The numeric value for the SOL is known in SI units, thus the value of the vertical leg of the wavelength formula multiplied by the value of the hypotenuse of the frequency triangle will give a known value. By varying the size of vertical leg of the wavelength triangle, either directly or by varying the triangle angle, will produce, through an iteration process, a physical size, in SI units, for the intrinsic wavelength at the scaling factor chosen for the intrinsic frequency.

Figure 5 denotes the numeric values of the wavelength and frequency legs, and the hypotenuses, when the angle represents the value associated with the time duration of the second. When expressed in SI units the relationships between the triangle pair

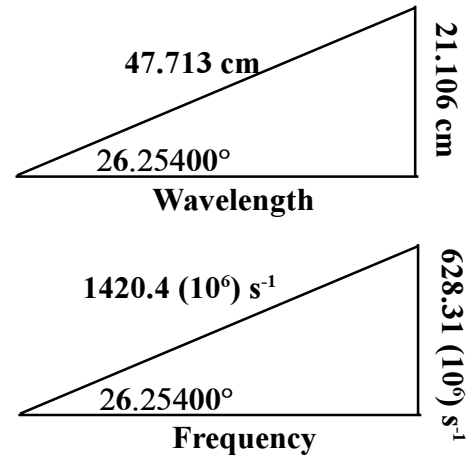


Figure 5.

are readily visualized.

The actual physical length of the scaled intrinsic wavelength was not known until it was extracted in a manner that allowed it to be compared to an existing system of measurement. The scaled intrinsic frequency is that of the precession emission of neutral hydrogen, usually referred to as the H1 hyperfine emission.

In SI units, the numeric value for the practical intrinsic wavelength, its associated frequency and the angle, can be calculated only to the precision permitted by that system.

The duration of the “unit of time” is a function of the angle. When the duration of the “unit of time” is assigned a value of “one” at 45 degrees, the time duration at other angles is given by the ratio of the cosecant values, equation (6).

$$R_{\tau} = \text{CSC}(\alpha)/\text{CSC}(45) \quad (6)$$

The duration of the second is approximately 1.5985... longer than τ .

IX. Conclusions

Identifying the intrinsic units was a key factor in developing the mathematical process, plus recognizing that wavelength and frequency can be presented in a geometric form, even though it is a special case.

Using intrinsic units, the geometric form and known physical law, frequency and wavelength are inversely proportional, the numeric value for the

speed of light was derived without knowing the physical values of the units by which it is defined.

The physical values for the units were mutually derived with the associated numeric value of $888.5765...(10^6) L/\tau$. It should not come as too much of a surprise that a unique characteristic of the hydrogen atom provides the common link between time, space and energy. The numeric value, when expressed in intrinsic units, provides a reference value for the velocity of electromagnetic waves, an energy level represented by the frequency of the H1 precession emission, and the number of electromagnetic cycles which define the duration of the time unit.

The mathematical precision of the size of the units will be nearly unlimited, and the precision, as applied in practice, will be limited only by the technology available at a given time.

The use of the mathematical form for the SOL value, $\sqrt{2*2\pi}(10^8)$, will make it convenient to use in general scientific formula, and its numeric value can be expressed to a level of precision suitable for the technology resultant desired. The mathematically derived reference value for the SOL provides a numeric form that is precisely suited for mathematical formulations used to determine the theoretical basis of physical law.

The real value of the SOL, and other frequencies of electromagnetic emission, can be expected to vary from the mathematically extracted value dependent upon the characteristics of the medium in which it is permitted to propagate.

The values derived by this process will allow the creation of a set of scientific units that are universal, permanent and invariant in time, with the base units referenced to the same numeric value, Figure 6.

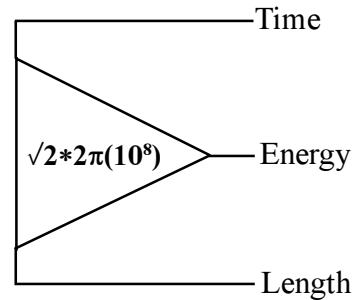


Figure 6 - Time, Energy, Length

The mathematical correlation between time, energy and length represents a “paradigm shift” in how scientific units of measure should be defined.

[1] <http://www.iupap.org/commissions/interunion/iu1/i1-2005.pdf>